Acoustic Instability: Influence of and on the Solid Phase

NORMAN W RYAN* AND RALPH L COATES†
Univei sity of Utah, Salt Lake City, Utah

In simple terms, acoustic instability in solid propellant rockets is observed as acoustic oscillations in the characteristic modes of the gas cavity driven by the combustion process at the gas-solid interface The effectiveness of the driving process, conveniently characterized by the effective acoustic admittance of the interface, may be significantly changed by transmission of acoustic energy into the solid Such changes can, in principle, be calculated from a knowledge of the viscoelastic properties of the solid Experiments employing 10-cm and longer grains in a T-burner show that relevant viscoelastic properties, inferred from burning data, can be used to calculate the solid phase contribution to the acoustic admittance The experiments show further that the viscoelastic properties change during the firing Apparently, the acoustic oscillations cause a large decrease in both the elastic modulus and the viscosity of the solid propellant

Nomenclature‡

a = viscoelastic parameter L^{-1}; imaginary part of ν
b = viscoelastic parameter, L^{-1}; real part of ν
c = speed of sound in solid, LT^{-1}; c_g = speed of sound in gas, LT^{-1}
E = acoustic energy content of gas, MT^{-2}
f = frequency of oscillation, cps
j = $(-1)^{1/2}$
k = $\frac{4}{3}$ viscosity, $ML^{-1}T^{-1}$
K = real part of specific acoustic admittance $Re(Y\rho c)$; K_f, of flame zone; K_i, of gas side; K , of solid referred to gas properties; K ', of solid referred to solid properties
l = x at gas-solid interface, L
M = elastic modulus, $ML^{-1}T^{-2}$
p = pressure $ML^{-1}T^{-2}$; p_g, gas phase at $x = p_{g0}$ at $t = 0$; p_0 solid phase at $x = 0$
t = time elapsed, T
u = acoustic particle velocity, LT^{-1}
$\dot{W}$ = rate transfer acoustic energy into gas phase MT^{-3}
x = axial distance measured into propellant from supported end L
Y = acoustic admittance, L^2TM^{-1}
α = oscillation amplitude growth constant, $d \ln |p| / dt$ T^{-1}
β = complex angular frequency, $\omega - j\alpha$ T^{-1}
ν = complex wave number, $b - ja$, L^{-1}
θ = phase difference between pressure signals at two ends of grain
ξ = acoustic particle displacement, L
ρ = density of solid, ML^{-3}; ρ_g, of gas, ML^{-3}
σ = imaginary part of specific acoustic admittance $Im(\rho c Y)$; σ ' referred to solid properties
τ = normal stress, $ML^{-1}T^{-2}$
$\varphi(t)$ = function of t defined by Eq (9), L
ω = angular frequency, $2\pi f$, T^{-1}

Introduction

THE extensive program of research on oscillatory combustion, also known as acoustic instability, was motivated by the observation that this kind of behavior is sometimes, though not always, followed by severe combustion irregularities McClure and Hart and their associates[2-4] proposed the first useful theoretical description of the phenomenon They proposed, as a part of their description, that the solid phase participates in the oscillations as a part of acoustical system

Experimental evidence of solid phase participation was obtained by Horton and reported by Ryan[5] and Horton [3] Additional evidence was obtained by Coates,[1] presented with a preliminary analysis by Ryan, Coates, and Baer,[6] and was presented with reduction to values of acoustic parameters by Coates [1] This paper extends that work to describe the behavior of the solid throughout the course of grain combustion It provides evidence that the properties of the solid can change markedly during burning

Related work is to be found chiefly from those concerned with fatigue failure of propellant under dynamic loading The results of Tormey and Britton,[7] whose paper refers also to other work, are of interest Loading laterally unsupported columns of composite propellants at $5g$ and frequencies near 100 cps, they found significant changes of properties in periods of many hours The change in the first hour was presumably measurable but not great Although the conditions we employed were mild as oscillatory combustion goes, they involved frequencies and loading levels an order of magnitude greater, and they produced significant changes in properties in a few seconds

Experimental

The apparatus employed is a side-vented end burner, also known as a T-burner, $1\frac{1}{2}$ in in diameter End-burning grains are placed in one or both ends against the end plates In runs employing a single long grain, the grain is backed by a piston (see Fig 1) During the firing, the piston is advanced automatically at the rate necessary to keep the burning surface at a fixed position This provision assures a constant length of gas column and, thereby, a constant oscillation frequency during the course of the entire run The pressure signals at both ends are sensed by Kistler Model 401 transducers and recorded on magnetic tape for subsequent playback and analysis

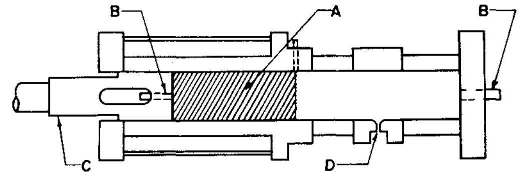

Fig 1 T-burner with piston drive: A, propellant grain; B, pressure transducers; C, piston; D, vent nozzle

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* Professor of Chemical Engineering Associate Fellow Member AIAA

† Now Technical Specialist, Lockheed Propulsion Co Redland, Calif Member AIAA

‡ Dimensions: M = mass L = length and T = time

The propellant employed in the tests reported here is called Utah F propellant It contains 80% by weight ammonium perchlorate, 18% PBAA copolymer with curing agent, and 2% copper chromite burning-rate catalyst Its density is 1 68 g/cm^3, and its normal burning rate is about 1 0 cm/sec at the conditions of test All tests reported here were carried out at 200 psi and at frequencies between 3000 and 5000 cps

Gas-Phase Considerations

We direct our attention from the gas side to the gas-solid interface in the burner (Fig 1) and note that the rate at which acoustic energy is pumped into the gas phase is the product of the real parts of the acoustic pressure and the acoustic velocity

$$Re(p_g)\ R_e(u) \tag{1}$$

Here velocity is taken positive in the direction from solid to gas The acoustic pressure is given by

$$p_g = |p_{g0}|e^{j(\beta t+\theta)}$$

where $\beta = \omega - j\alpha$; the velocity is given by $u = Yp_g$, where Y is the complex acoustic admittance,

$$Y = [1/(\rho c)_g](K + j\sigma)$$

When the product of real parts of u and p is formed, Eq (1) integrated over the period of a complete cycle, and the result multiplied by the frequency, we have the time average energy input

$$\dot{W} = (K/2)[|p_g|^2/(\rho c)_g]$$

The assumption that $\alpha/f \ll 1$ has been made

The acoustic energy content of the gas per unit cross-sectional area, if the length of the gas column is one-half wavelength (oscillation in the axial fundamental), is

$$E = (1/8f)[|p_g|^2/(\rho c)_g] \tag{2}$$

The rate of the fractional increase due to the process at the interface is then $\dot{W}/E = 4fK$ The rate of fractional increase of acoustic energy is obtained by differentiating Eq (2) to give

$$\dot{E}/E = 2(d \ln|p_g|/dt) = 2\alpha$$

The quantities $\dot{W}$ and $\dot{E}$ are not the same, as the latter includes effects of gains and losses in all parts of the system They can be considered the same, however, if K is interpreted as containing terms to accommodate gains and losses at places other than the interface Then,

$$K = \alpha/2f \tag{3}$$

where

$$K = K_f + \Sigma K_i + K_s \tag{4}$$

In Eq (4), K_f is the contribution of the chemical and transport processes at the interface, ΣK_i is the sum of contributions in the gas phase and at the walls, and K_s is the contribution of the solid phase and its supporting boundaries

One method, devised by Coates[1] to obtain K_f, is to burn short grains for which K is negligible In one experiment, one short grain is burned in one end of the burner, and $\alpha = \alpha_1$ is measured; in a second experiment, short grains are placed in both ends and burned at the same oscillation frequency, and $\alpha = \alpha_2$ is measured It is assumed that K_f and ΣK_i are the same in both experiments Then Eqs (3) and (4) give

$$K_f = (\alpha_2 - \alpha_1)/2f \qquad \Sigma K_i = (2\alpha_1 - \alpha_2)/2f$$

When K is desired, a similar procedure is used; however, in the second experiment a single long grain is burned in one end and $\alpha = \alpha_l$ measured This time Eqs (3) and (4) give

$$K = (\alpha_l - \alpha_1)/2f \tag{5}$$

The condition for suppression of oscillations by transfer of acoustic energy to the solid phase is seen to be $\alpha_l = 0$, or

$$K_s \leqslant -(\alpha_1/2f) \tag{6}$$

As we have reported previously,[6] when a long grain is burned, there may be two periods of oscillation separated by a period of no oscillation during which the forementioned condition obtains The stable period does not always occur, or the intermediate period may be one of reduced oscillation amplitude In our burner, when Utah F propellant undergoes oscillatory burning at the frequency of 4300 cps, $\alpha_1/2f$ is 0 0065

Solid-Phase Considerations

We consider now the oscillations in the solid phase, assuming that the acoustic particle velocity, its derivatives and the derivatives of density, and the normal stress are small quantities We assume also that radial and tangential variations in all quantities are negligible No defense is offered for this last assumption beyond the pragmatic observation that the analysis, so simplified, will still illustrate the important features of solid phase oscillation The force balance yields

$$\rho(\partial u/\partial t) = \partial\tau/\partial x$$

The normal stress is assumed to have elastic and viscous terms:

$$\tau = M\xi_x + k\xi_{tx} \tag{7}$$

where M is the elastic modulus that pertains to the system considered, and k is recognized as four-thirds the viscosity Subscripts x and t indicate partial differentiation Substitution of the expression for normal stress into the force balance gives the wave equation

$$\rho\xi_{tt} = M\xi_{xx} + k\xi_{txx} \tag{8}$$

The following solution is assumed for the wave equation:

$$\xi = \varphi(t)\ \sinh j\nu x \tag{9}$$

where $\nu = b - ja$ This solution satisfies the boundary conditions that ξ and ξ_t must be zero at $x = 0$, the rigid end plate The function of time $\varphi(t)$ is determined by equating the stress expression to the observed stress (negative acoustic pressure) at $x = 0$:

$$\tau_0 = |\tau_{00}|e^{j\beta t} = j\nu(M\varphi + k\varphi')$$

The equation is solved for $\varphi(t)$, and Eq (9) is rewritten as

$$\xi = \frac{|\tau_{00}|}{j\nu(M + j\beta k)}\, e^{j\beta t} \sinh j\nu x \tag{10}$$

Substitution of Eq (10) into Eq (8) gives

$$M + j\beta k = \rho(\beta/\nu)^2$$

which, substituted into Eq (10), gives

$$\xi = -(j\nu|\tau_{00}|/\rho\beta^2)e^{j\beta t} \sinh j\nu x \tag{11}$$

Then

$$\tau = |\tau_{00}|e^{j\beta t} \cosh j\nu x \tag{12}$$

Equations (11) and (12) are the basic equations employed below, supplemented by one other It is observed that the complex speed of sound in the solid is given by $\tilde{c} = \beta/\nu$, the absolute value being given by

$$c = (\omega/b)\{[1 + (\alpha/\omega)^2]/[1 + (a/b)^2]\}^{1/2} \tag{13}$$

The acoustic admittance is given by

$$Y = (1/\rho c)(K' + j\sigma') = -(\xi_t/\tau)$$

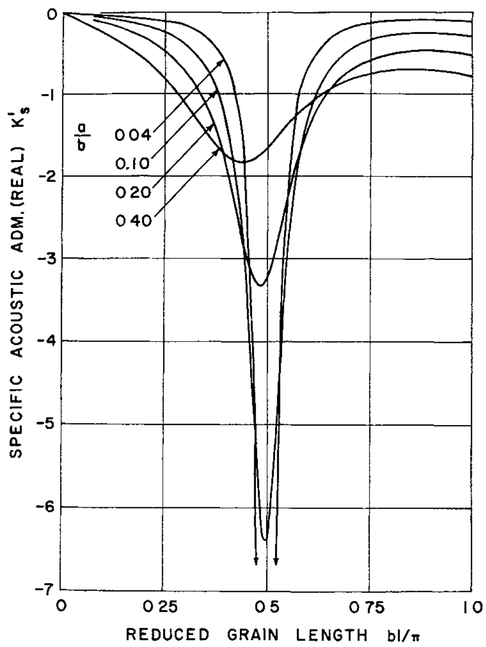

Fig 2 Real part of the specific acoustic admittance of the solid grain as a function of grain length and viscoelastic parameters [see Eq (14)]

Application of Eqs (11–13) gives, for $x = l$,

$$K_s' = -\frac{\frac{1}{2}\left\{\left(1 + \frac{\alpha}{\omega}\frac{a}{b}\right)\sinh 2al - \left(\frac{\alpha}{\omega} - \frac{a}{b}\right)\sin 2bl\right\}}{\left[1 + \left(\frac{\alpha}{\omega}\right)^2\right]^{1/2}\left[1 + \left(\frac{a}{b}\right)^2\right]^{1/2}(\sinh^2 al + \cos^2 bl)} \tag{14}$$

$$\sigma_s' = -\frac{\frac{1}{2}\left\{\left(1 + \frac{\alpha}{\omega}\frac{a}{b}\right)\sin 2bl + \left(\frac{\alpha}{\omega} - \frac{a}{b}\right)\sinh 2al\right\}}{\left[1 + \left(\frac{\alpha}{\omega}\right)^2\right]^{1/2}\left[1 + \left(\frac{a}{b}\right)^2\right]^{1/2}(\sinh^2 al + \cos^2 bl)} \tag{15}$$

Evaluation of the parts of specific acoustic admittance requires determination of l, α, ω, a, and b Of these, grain length can be closely estimated, and frequency is determined by gas-phase geometry and the hot-gas properties The value of α is related to K by Eqs (3) and (4) Fortunately, it is usually the case that $\alpha \ll \omega$ so that α need not be known Figure 2 shows K' as a function of bl and a/b for the case of $\alpha/\omega = 0$

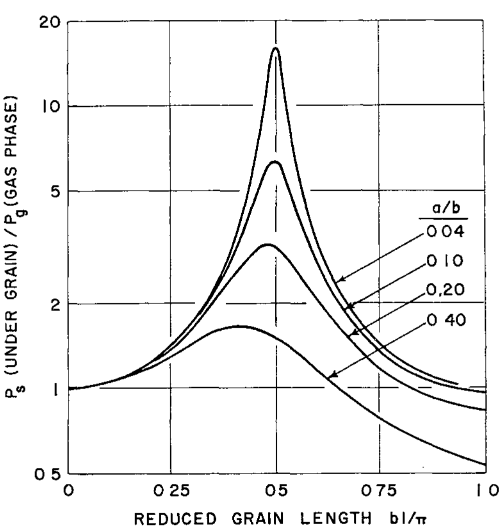

Fig 3 Ratio of acoustic pressure amplitudes, supported end to free end of grain, as a function of grain length and viscoelastic parameters [see Eq (16)]

Independent determination of a and b is very difficult As shown below, both constants change considerably during the course of a firing In the experiments discussed in this paper, two additional measurements were made: the acoustic pressure in the gas phase and the phase difference between the pressure signals at the ends of the chamber These allow calculation of a and b as shown in the following

The stress at the free end of the grain is given by Eq (12),

$$\tau_l = |\tau_{00}| e^{j\beta t} \cosh j\nu l$$

and also by

$$\tau_l = |\tau_{l0}| e^{j(\beta t + \theta)}$$

When continuity of normal stress across the free face is assumed, and it is observed that α is the same in both phases, then

$$\frac{|\tau_{00}|}{|\tau_{l0}|} = \frac{|\tau_0|}{|\tau_l|} = \frac{|p_0|}{|p_l|} = \frac{1}{(\sinh^2 al + \cos^2 bl)^{1/2}} \tag{16}$$

and

$$\tan\theta = \tanh al \tan bl \tag{17}$$

Equations (16) and (17) give a and b implicitly as functions of θ and the ratio of pressure amplitudes These equations are plotted on Figs 3 and 4 Two comments are in order First, it has been assumed that the gas-phase pressure amplitude is the same at both ends of the gas column This is a safe assumption, as the ρc ratio, solid to gas, is on the order of 1000 and $\alpha \ll \omega$ Second, the phase angle θ differs by π from the observed phase angle between signals recorded at the extreme ends of the chamber

Analysis of Data

The equations just developed have been applied to Coates' data Corresponding values of frequency, grain length, phase angle, and pressure amplitude ratio were used with more detailed versions of Figs 3 and 4 in order to obtain values of a

Table 1 Changes in viscoelastic properties during firing

	Length grain, cm	b, cm^{-1}	a, cm^{-1}	Speed of sound cm/sec
Run 295:				
60 C, 200 psi	11 9	0 18	0 021	153 000
ω = 28 300 ±	10 2	0 19	0 016	146 000
1300 rad/sec	8 5	0 19	0 024	145 000
		8–6, no oscillation		
	6 1	0 23	0 038	123 000
	5 6	0 21	0 050	132 000
	3 9	0 24	0 075	113 000
	2 2	0 38	0 101	72 000
Run 293:				
60 C, 200 psi	9 8	0 22	0 012	139 000
ω = 30 000 ± 1500	9 0	0 20	0 013	147 000
rad/sec	8 0	0 25	0 024	121 000
	7 7	0 25	0 038	118 000
	7 2	0 23	0 079	123 000
	5 2	0 31	0 17	119 000
	4 1	0 26	0 14	105 000
	2 9	0 38	0 23	68 000
	1 2	0 96	0 49	28 000
Run 285:				
0 C 200 psi	10 2	0 18	0 043	154 000
ω = 28,500 ± 2000	8 5	0 19	0 067	140 000
rad/sec	6 9	0 21	0 089	126 000
	5 2	0 25	0 12	103 000
	3 0	0 26	0 23	83 000

and b The most striking result obtained is the marked increase in the values of both of these viscoelastic parameters during the course of a firing Table 1 presents the results for three of the ten firings for which suitable data have been obtained Typically, in the course of a 10- to 12-sec firing (grain initially 10–12 cm long), the value of b increased by a factor between 2 and 3, and that of a by a factor between 5 and 20 The ratio of a/b increases with a corresponding reduction in the Q of the system

We have not been able to correlate these parameters with either initial grain temperature or frequency It is only by selecting from a considerable volume of data taken for grains initially at 30°C that we can estimate initial values of b and a At an angular frequency of 19,000 rad/sec, the initial value of b is 0 10 ± 0 02 cm^{-1} The corresponding values of a vary from 0 003–0 02 cm^{-1} The initial speed of sound in the propellant is then on the order of 200,000 cm/sec according to Eq (13) After the propellant has been massaged by the oscillations, its speed of sound appears to drop to less than that of the gas which is 93,000 cm/sec

Figure 5 is a replot of Fig 2 with al and bl as coordinates and K' as parameter A parametric line encloses a portion of the field such that, if K' for the line corresponds to $K_s = -\alpha_1/2f$ [see Eq (6)], then systems whose states fall in the enclosed area are stable to acoustic oscillation If, in a firing, the values of a and b do not change, the path of the firing is from right to left along a straight line passing through (0,0) Acoustic oscillations will be observed when the path is outside the closed region; they cease when the path is inside that region

Paths for the three firings of Table 1 are shown in Fig 5 Run 295 appeared to proceed as expected When bl/π reached about 0 53, oscillations ceased The oscillations began again when bl/π was about 0 45 The critical value of K' was apparently about -4, greater than -6 The path after oscillations resumed indicates sharp changes in a and b, suggesting a sudden breakdown in the internal structure of the solid

Run 293 was the twin of run 295, differing, it was supposed, only in the initial grain length In this run, however, oscillations continued uninterrupted until burnout The sudden change in viscoelastic properties occurred before K_s' dropped to the critical value, and the path was detoured around the zone of no oscillation Similar behavior is inferred for run 285, though it is likely that, had a and b remained unchanged, oscillations would not have died out but would merely have diminished momentarily as the path skimmed by the zone of no oscillations

The critical value of K_s, the least value permitting oscillations, can be calculated from the observed critical K' for run 295 This is seen from Fig 5 to be between -4 and -6 We take $(\rho c)_g$ as 200 g/cm²/sec, c as 135,000 cm/sec (run 295), ρ as 1 68 g/cm³, and find

$$K = \frac{(\rho c)_g}{\rho c} K_s' = \frac{200}{(1\ 68)(135{,}000)} \begin{Bmatrix} -4 \\ -6 \end{Bmatrix} = \begin{Bmatrix} -0\ 0035 \\ -0\ 0053 \end{Bmatrix}$$

The value calculated from gas-phase considerations is $-0\ 0065$, as pointed out in the discussion following Eq (6) The discrepancy, greater than known errors, is attributed to a reduction in the ΣK_i term of Eq (4) The value of α_1 is measured at the start of a run when the chamber is clean, whereas the value of K' is determined after several seconds of burning when the walls are dirty In support of this view is the observation that frequently oscillations die out before the end of a run when K_s is approaching zero In run 295, for example, oscillations ceased a full second before burnout There was no evidence of irregular behavior The chamber pressure trace continues flat to its termination in a sharp tailoff

In view of the assumptions required for the theoretical model adopted, it is rash to attribute much significance to the modulus M and the viscosity ($\frac{3}{4}$ of k) Nevertheless, it is of

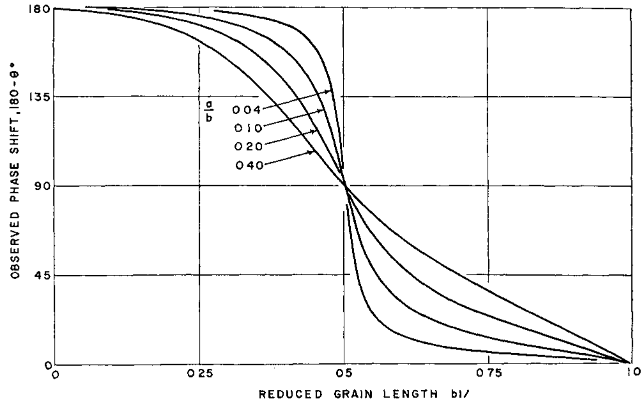

Fig 4 Difference in pressure signal phase θ, between supported and free ends of the grain, as a function of grain length and viscoelastic parameters [see Eq (17)] The observed gas phase signal is observed at the end plate and differs from θ at gas-solid interface by 180

interest to estimate their values This can be done if M and k are considered real When Eqs (11) and (13) are employed with Eq (8), we get, if α is very small compared to ω,

$$M = \rho c^2 \frac{1 - (a/b)^2}{1 + (a/b)^2}$$

$$k = \frac{2\rho c a}{b^2}\left[1 + \left(\frac{a}{b}\right)^2\right]^{-3/2}$$

For the case of fresh unworked propellant, at the conditions of the firing reported here, the modulus is of the order of 10^6 psi, and the viscosity is of the order of 500 kpoises For the worked propellant, the modulus is of the order of 10^5 psi and the viscosity is about 150 kpoises It is reasonable to suppose that the true modulus and viscosity undergo about the same relative changes

To this point, the discussion implies the assumption that the propellant deteriorates uniformly, an unlikely possibility The computed values of a and b after the initial part of a firing are probably average values, local values varying greatly from place to place The question of what part of the grain softens first or most cannot be answered at this time We can, however, make the following observations about the early stages of a firing If the rate of degeneration is a function of displacement ξ or acceleration ξ_{tt}, it will be greatest at distances of odd quarter wavelengths from the supported end If the rate is a function of strain ξ_x or rate of strain ξ_{xt},

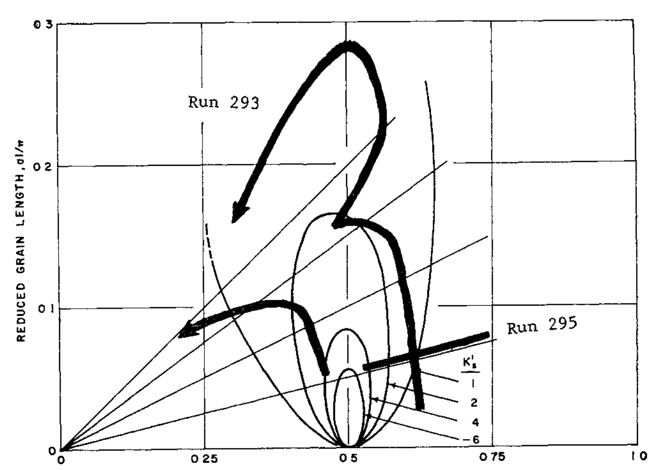

Fig 5 Firing path diagram showing lines of constant solid phase acoustic admittance The path for a grain of constant properties is a straight line through (0,0) Actual paths for two firings are shown

it will be greatest at the supported end and multiples of a half wavelength from there In either case, softening can take place at positions in the body of the grain at some distance from the burning surface

Conclusions

Ad hoc viscoelastic parameters have been defined in a one-dimensional, linearized analysis of acoustic instability in T-burners Their values, computed from experimental data, show an effect that has not been previously demonstrated, namely, that acoustic instability can markedly change viscoelastic properties during a firing The change is in the direction of reducing the elasticity and the viscosity of the solid propellant In our experiments, the change was brought about by normal stress amplitudes rarely, if ever, as great as 50 psi Presumably, shear stresses were no greater As the acoustic energy pumped into the system was not sufficient to raise the temperature significantly, we infer that mechanical working of the solid produced the changes

The lesson is clear If acoustic instability at such apparently innocent levels of severity can soften at least some propellants, then either susceptible propellants, marginal in their viscoelastic properties, should not be used, or acoustic instability must be avoided We speculate that propellant softening is the key first step in the mechanism whereby acoustic instability sometimes provokes more serious combustion irregularities

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